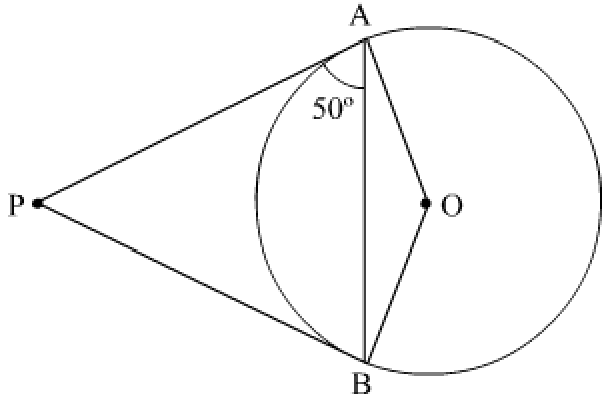
Section A

Q1 From an external point P, tangents PA and PB are drawn to a circle with centre O. If ∠PAB = 50°, then find $\angle AOB$.

Ans:-



It is given that PA and PB are tangents to the given circle.

$\therefore \angle PAO = 90^\circ$ (Radius is perpendicular to the tangent at the point of contact.)

Now,

$\angle PAB = 50^\circ$ (Given)

$\therefore \angle OAB = \angle PAO - \angle PAB = 90^\circ - 50^\circ = 40^\circ$

In ΔOAB,

OB = OA (Radii of the circle)

$\therefore \angle OAB = \angle OBA = 40^\circ$ (Angles opposite to equal sides are equal.)

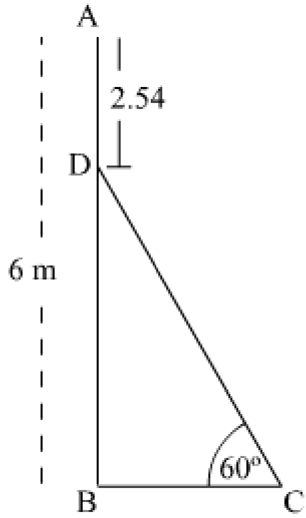
Now,

$\angle AOB + \angle OAB + \angle OBA = 180^\circ$ (Angle sum property)

$\Rightarrow \angle AOB = 180^\circ - 40^\circ - 40^\circ = 100^\circ$

Q2 In Fig. 1, AB is a 6 m high pole and CD is a ladder inclined at an angle of 60° to the horizontal and reaches up to a point D of pole. If AD = 2.54 m,

find the length of the ladder. (use3√=1.73)



Ans:- In the given figure,
AB = AD + DB = 6 m
Given: AD = 2.54 m
⇒ 2.54 m + DB = 6 m
⇒ DB = 3.46 m
Now, in the right triangle BCD,

$$\frac{BD}{CD} = \sin 60^\circ$$

$$\Rightarrow \frac{3.46\,m}{CD} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{3.46\,m}{CD} = \frac{1.73}{2}$$

$$\Rightarrow CD = \frac{2 \times 3.46\,m}{1.73}$$

$$\Rightarrow CD = 4\,m$$

Thus, the length of the ladder CD is 4 m.

Q3 Find the 9th term from the end (towards the first term) of the A.P. 5, 9, 13, ..., 185.
Ans:- Common difference, *d*, of the AP = 9 − 5 = 4

Last term, *l*, of the AP = 185
We know that the n^{th} term from the end of an AP is given by $l - (n - 1)d$.
Thus, the 9^{th} term from the end is
185 − (9 − 1)4
= 185 − 4 × 8
= 185 − 32
= 153

Q4 Cards marked with number 3, 4, 5, ..., 50 are placed in a box and mixed thoroughly. A card is drawn at random form the box. Find the probability that the selected card bears a perfect square number.
Ans:- It is given that the box contains cards marked with numbers 3, 4, 5, ..., 50.
∴ Total number of outcomes = 48
Between the numbers 3 and 50, there are six perfect squares, i.e. 4, 9, 16, 25, 36 and 49.
∴ Number of favourable outcomes = 6
∴ Probability that a card drawn at random bears a perfect square

$$= \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes} = \frac{6}{48} = \frac{1}{8}$$

SECTION B

Q5 If $x = \frac{2}{3}$ and $x = -3$ **are roots of the quadratic equation** $ax^2 + 7x + b = 0$ **, find the values of *a* and *b*.**

Ans:- The given equation is $ax^2 + 7x + b = 0.$ Its roots are given as -3 and $\frac{2}{3}.$
Now,

$$Sum\ of\ the\ roots = \frac{-(Coefficient\ of\ x)}{Coefficient\ of\ x^2}$$

$\Rightarrow -3+\frac{2}{3}=\frac{-(7)}{A}$

$\Rightarrow \frac{-9+2}{3}=\frac{-7}{a}$

$\Rightarrow \frac{-7}{3}=\frac{-7}{a}$

$\Rightarrow a=3$

Also,

Product of the roots = $\frac{Constant\ term}{Coefficient\ of\ x^2}$

$\Rightarrow -3\times\frac{2}{3}=\frac{b}{a}$

$\Rightarrow -2=\frac{b}{3}$

$\Rightarrow b=-6$

Thus, the values of *a* and *b* are 3 and −6, respectively.

Q6 Find the ratio in which *y*-axis divides the line segment joining the points A(5, −6) and B(−1, −4). Also find the coordinates of the point of division.

Ans:- Let (0, *α*) be a point on the *y*-axis dividing the line segment AB in the ratio *k* : 1.

Now, using the section formula, we get

$(0,\alpha)=\left(\frac{-k+5}{k+1},\frac{4k-6}{k+1}\right)$

$\Rightarrow \frac{-k+5}{k+1}=0,$

$-4k-6k+1=\alpha$

Now,

$\frac{-k+5}{k+1}=0$

$\Rightarrow -k+5=0$

$\Rightarrow k=5$

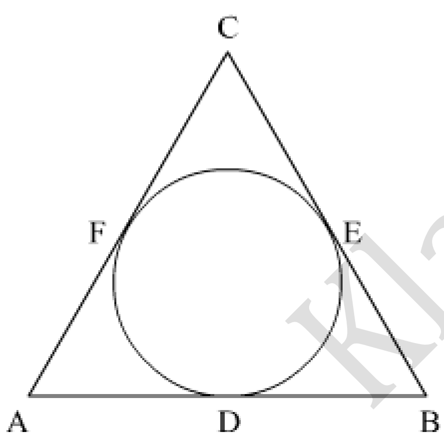
Also,

$$\frac{4k-6}{k+1}=\alpha$$
$$\Rightarrow \frac{-4\times 5-6}{5+1}=\alpha$$
$$\Rightarrow \alpha=\frac{-26}{6}$$
$$\Rightarrow \alpha=-\frac{13}{3}$$

Thus, the *y*-axis divides the line segment in the ratio *k* : 1, i.e. 5 : 1.

Also, the coordinates of the point of division are (0, *α*), i.e. $\left(0,-\frac{13}{3}\right)$.

Q7 In Fig. 2, a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA and 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF.



Ans:- It is given that
AB = 12 cm
⇒ AD + BD = 12 cm .....(1)
BC = 8 cm
⇒ BE + CE = 8 cm .....(2)
CA = 10 cm
⇒ AF + CF = 10 cm .....(3)
CF and CE act as tangents to the circle from the external point C.
It is known that the lengths of tangents drawn from an external point to a circle are equal.
$\therefore CF = CE \quad ...(4)$

∴ CF = CE .....(4)
Similarly, AF and AD act as tangents to the circle from the external point A.
∴ AF = AD .....(5)
Also, BD and BE act as tangents to the circle from the external point B.
∴ BD = BE .....(6)
Using (4) and (2), we get
BE + CF = 8 cm .....(7)
Using (5) and (3), we get
AD + CF = 10 cm .....(8)
Using (6) and (1), we get
AD + BE = 12 cm .....(9)
Adding (7), (8) and (9), we get
BE + CF + AD + CF + AD + BE = 8 cm + 10 cm + 12 cm
⇒ 2AD + 2BE + 2CF = 30 cm
⇒ 2(AD + BE + CF) = 30 cm
⇒ AD + BE + CF = 15 cm .....(10)
Subtracting (7) from (10), we get
AD + BE + CF − BE − CF = 15 cm − 8 cm
⇒ AD = 7 cm
Subtracting (8) from (10), we get
AD + BE + CF − AD − CF = 15 cm − 10 cm
⇒ BE = 5 cm
Subtracting (9) from (10), we get
AD + BE + CF − AD − BE = 15 cm − 12 cm
⇒ CF = 3 cm
Thus, the lengths of AD, BE and CF are 7 cm, 5 cm and 3 cm, respectively.

Q8 The *x*-coordinate of a point P is twice its *y*-coordinate. If P is equidistant from Q(2, –5) and R(–3, 6), find the coordinates of P.
Ans:-Let the *y*-coordinate of the point P be *a*.
Then, its *x*-coordinate will be 2*a*.
Thus, the coordinates of the point P are (2*a*, *a*).
It is given that the point P (2*a*, *a*) is equidistant from Q (2, −5) and R (−3, 6).
Thus, we have

$$\sqrt{(2a-2)^2+(a-(-5)^2)} = \sqrt{(2a-(-3)^2+(a-6)^2}$$

$$\Rightarrow \sqrt{(2a-2)^2+(a+5)^2} = \sqrt{(2a+3)^3+(a-6)^2}$$

$$\Rightarrow \sqrt{4a^2+4-8a+a^2+25+10a}=\sqrt{4a^2+9+12a}$$

$$\Rightarrow \sqrt{5a^2+2a+29}=\sqrt{5a^2+45}$$

Squaring both sides, we get

$$5a^2+2a+29=5a^2+45$$

$$\Rightarrow 5a^2+2a-5a^2=45-29$$

$$\Rightarrow 2a=16$$

$$\Rightarrow a=8$$

Thus, the coordinates of the point P are (16, 8), i.e. (2 × 8, 8).

Q9 How many terms of the A.P. 18, 16, 14, ... be taken so that their sum is zero?

Ans:- The given AP is 18, 16, 14, ...
First term of the AP = 18
Common difference = 16 − 18 = −2
Let the sum of the first x terms of the AP be 0.

Sum of the first x terms = $\frac{x}{2}\left[2\times18+(x-1)(-2)\right]=0$

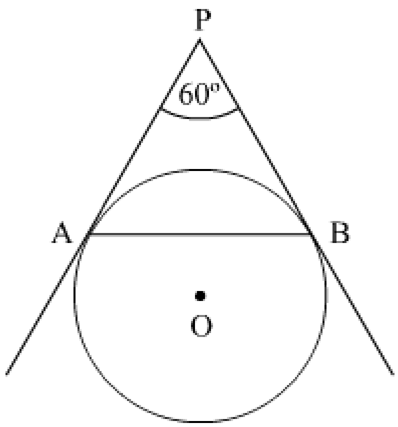
$$\Rightarrow \frac{x}{2}\left[36+(-2x+2)\right]=0$$

$$\Rightarrow x(36-2x+2)=0$$

$$\Rightarrow x(38-2x)=0$$

Now, either x = 0 or 38 − 2x = 0
But the number of terms cannot be 0.
∴ 38 − 2x = 0
⇒ 38 = 2x
⇒ x = 19
Thus, the sum of the first 19 terms of the AP is 0.

Q10 In Fig. 3, AP and BP are tangents to a circle with centre O, such that AP = 5 cm and $\angle APB = 60^\circ$. Find the length of chord AB.

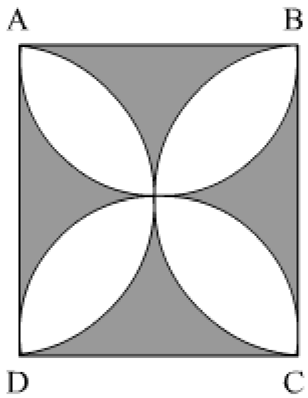


Ans:- PA and PB are tangents drawn to the given circle from an external point P.
It is known that the lengths of the tangents drawn from an external point to a circle are equal.
∴ PA = PB
In ΔPAB , sides PA and PB are of the same length.
Hence, ?PAB is isosceles, with PA = PB and $\angle PAB = \angle PBA = x(say).$
It is given that
$\angle APB = 60°$
We know that the sum of the angles of a triangle is 180°.
In ΔPAB,
$\angle PAB + \angle PBA + \angle APB = 180°$
∴ $x + x + 60° = 180°$
⇒ $2x = 120°$
⇒ $x = 60°$
Thus,
$\angle PAB = \angle PBA = \angle APB = 60°$
Since all angles of ?PAB are of the same measure, ΔPAB is equilateral, with AP = BP = AB.
It is given that
AP = 5 cm
∴ AB = AP = 5 cm
Thus, the length of the chord AB is 5 cm.

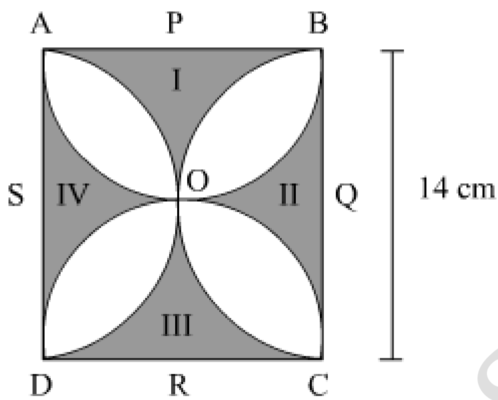
SECTION C

Q11 In Fig. 4, ABCD is a square of side 14 cm. Semi-circles are drawn with each side of square as diameter. Find the area of the shaded region. (

$use\ \pi = \frac{22}{7}$)



Ans:-



Let the four shaded regions be I, II, III and IV and the centres of the semicircles be P, Q, R and S, as shown in the figure.
It is given that the side of the square is 14 cm.
Now,
Area of region I + Area of region III = Area of the square − Areas of the semicircles with centres S and Q.

$= 14 \times 14 - 2 \times \frac{1}{2} \times \pi \times 7^2$ (? Radius of the semicircle=7 cm)

$= 196 - 49 \times \frac{22}{7}$

=196−154

$= 42\,cm^2$

Similarly,
Area of region II + Area of region IV = Area of the square − Areas of the

semicircles with centres P and R.

$=14\times 14-2\times\frac{1}{2}\times\pi\times 7^2$ (? Radius of the semicircle=7 cm)

$=196-49\times\frac{22}{7}$

=196−154

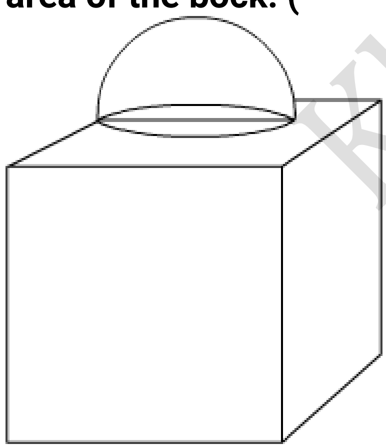
$=42\,cm^2$

Thus,
Area of the shaded region = Area of region I + Area of region III + Area of region II + Area of region IV

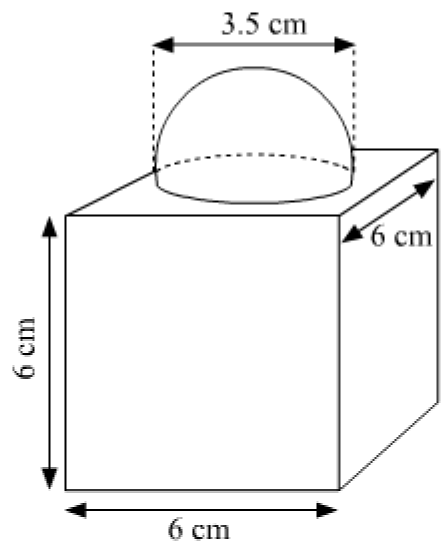
$=42\ cm^2+42\ cm^2$

$=84cm^2$

Q12 In Fig. 5, is a decorative block, made up two solids – a cube and a hemisphere. The base of the block is a cube of side 6 cm and the hemisphere fixed on the top has diameter of 3.5 cm. Find the total surface area of the bock. ($use\ \pi=\frac{22}{7}$)



Ans:-



Surface area of the block = Total surface area of the cube – Base area of the hemisphere + Curved surface area of the hemisphere

$= 6\times\left(Edge\right)^2 - \pi r^2 + 2\pi r^2$

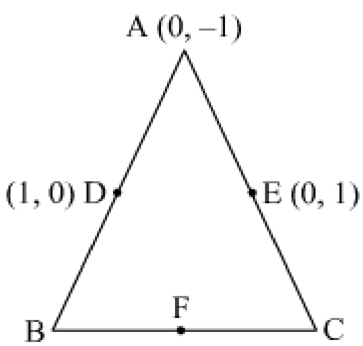
$=\left(6^3 + \pi r^2\right)$

$=\left(216+\frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2}\right)$

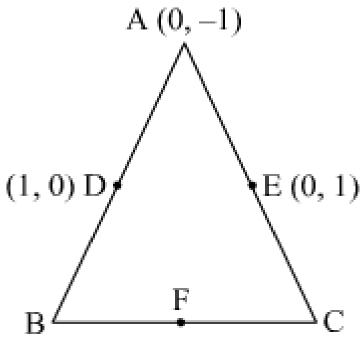
=(216+9.625)

$= 225.625 cm^2$

Q13 In Fig. ABC is a triangle coordinates of whose vertex A are (0, −1). D and E respectively are the mid-points of the sides AB and AC and their coordinates are (1, 0) and (0, 1) respectively. If F is the mid-point of BC, find the areas of ΔABC *and* ΔDEF.



Ans:-



Let the coordinates of B and C be (x_2, y_2) *and* (x_3, y_3), respectively.

D is the midpoint of AB.
So,

$$(1, 0) = \left(\frac{x_2 + 0}{2}, \frac{y_2 - 1}{2}\right)$$

$$\Rightarrow 1 = \frac{x_2}{2} \; and \; 0 = \frac{y_2 - 1}{2}$$

$$\Rightarrow x_2 = 2 \; and \; y_2 = 1$$

Thus, the coordinates of B are (2, 1).
Similarly, E is the midpoint of AC.
So,

$$(0, 1) = \left(\frac{x_3 + 0}{2}, \frac{y_3 - 1}{2}\right)$$

$$\Rightarrow 0 = \frac{x^3}{2} \; and \; 1 = \frac{y_3 - 1}{2}$$

$$\Rightarrow x^3 = 0 \quad and \; y^3 = 3$$

Thus, the coordinates of C are (0, 3).
Also, F is the midpoint of BC. So, its coordinates are

$$\left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$$

Now,
Area of a triangle

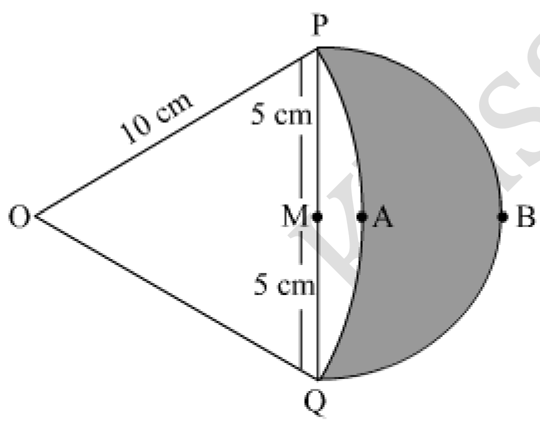
$$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

Thus, the area of ΔABC is

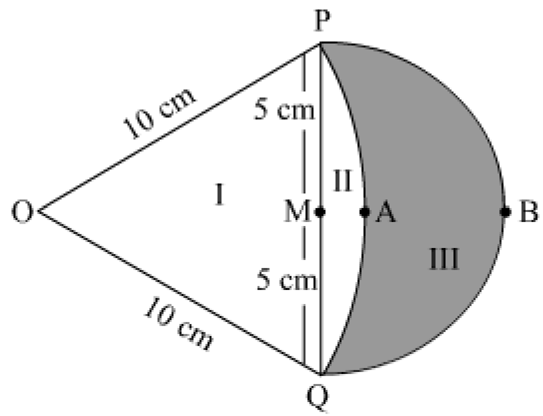
$$\frac{1}{2}\left[0(1-3)+2(3+1)+0(-1-1)\right]$$
$$=\frac{1}{2}\times 8$$
= 4 square units
And the area of ΔDEF is

$$\frac{1}{2}\left[1(1-2)+0(2-0)+1(0-1)\right]$$
$$=\frac{1}{2}\times(-2)$$
=1 square unit (Taking the numerical value, as the area cannot be negative)

Q14 In Fig. 7, are shown two arcs PAQ and PBQ. Arc PAQ is a part of circle with centre O and radius OP while arc PBQ is a semi-circle drawn on PQ ad diameter with centre M. If OP = PQ = 10 cm show that area of shaded region is $25\left(\sqrt{3}-\frac{\pi}{6}\right)cm^2$ **.**



Ans:- Given: OP = OQ = 10 cm
It is known that tangents drawn from an external point to a circle are equal in length.
So,
OP = OQ = 10 cm



Therefore, ΔABC is an equilateral triangle.

$\Rightarrow \angle POQ = 60°$

Now,

Area of part II = Area of the sector − Area of the equilateral triangle POQ

$$= \frac{\angle POQ}{360°} \times \pi r^2 - \frac{\sqrt{3}}{4} \times (10)^2$$

$$= \frac{60°}{360°} \times \pi (10^2) - \frac{\sqrt{3}}{4} \times (10)^2$$

$$= 100\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$$ sq units

Area of the semicircle on diameter PQ = Area of part II + Area of part III

$$= \frac{1}{2} \times \pi (5)^2 = \frac{25}{2} \pi \, sq \, units$$

∴ Area of the shaded region (part III)

$$= \frac{25}{2}\pi - 100\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$$

$$= \frac{25}{2}\pi - \frac{100}{6}\pi + 25\sqrt{3}$$

$$= 25\sqrt{3} - \frac{25}{6}\pi$$

$$= 25\left(\sqrt{3} - \frac{\pi}{6}\right) sq \, units$$

Hence proved.

Q15 If the sum of first 7 terms of an A.P. is 49 and that of its first 17 terms is 289, find the sum of first *n* terms of the A.P.

Ans:- Let the first term and the common difference of the given AP be *a* and *d*, respectively.

Sum of the first 7 terms, S_7 = 49

We know

$$S = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow \frac{7}{2}(2a + 6d) = 49$$

$$\Rightarrow \frac{7}{2} \times 2(a + 3d) = 49$$

$$\Rightarrow a + 3d = 7 \quad .....(1)$$

$$S_{17} = 289$$

Sum of the first 17 terms,

$$\Rightarrow \frac{17}{2}(2a + 16d) = 289$$

$$\Rightarrow \frac{17}{2} \times 2(a + 8d) = 289$$

$$\Rightarrow a + 8d = \frac{289}{17} = 17$$

$$\Rightarrow a + 8d = 17 \quad .....(2)$$

Subtracting (2) from (1), we get

$$5d = 10$$

$$\Rightarrow d = 2$$

Substituting the value of *d* in (1), we get

a = 1

Now,

Sum of the first *n* terms is given by

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[2 \times 1 + 2(n-1)\right]$$

$$= n(1 + n - 1) = n^2$$

Therefore, the sum of the first *n* terms of the $AP \text{ is } n^2$.

Q16 Solve for *x*:

$$\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)}=0,$$
$$x\neq 3,-3/2$$

Ans:- Given:

$$\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)}=0,$$
$$x\neq 3,-3/2$$
$$\Rightarrow \frac{2x(2x+3)+(x-3)+3x+9}{(x-3)(2x+3)}=0$$

$$\Rightarrow 4x^2+6x+x-3+3x+9=0$$
$$\Rightarrow 4x^2+10x+6=0$$
$$\Rightarrow 4x^2+4x+6x+6=0$$

$$\Rightarrow 4x(x+1)+6(x+1)=0$$
$$\Rightarrow (x+1)(4x+6)=0$$

$$\Rightarrow x+1=0 \text{ or } 4x+6=0$$
$$\Rightarrow x=-1,-\frac{3}{2}$$

But $x\neq -\frac{3}{2}$

Thus, $x=-1$ is the solution of the given equation.

Q17 A well of diameter 4 m is dug 21 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 3 m to form an embankment. Find the height of the embankment.

Ans:- Let *r* and *h* be the radius and depth of the well, respectively.

$$\therefore r=\frac{4}{2}=2\,m \text{ and } h=21\,m$$

Let *R* and *H* be the outer radius and height of the embankment, respectively.

∴ $R = r + 3 = 2 + 3 = 5$ m

Now,

Volume of the earth used to form the embankment = Volume of the earth dug out of the well

$\pi\left(R^2-r^2\right)H=\pi r^2h$

$\Rightarrow H=\frac{r^2h}{R^2-r^2}$

$\Rightarrow H=\frac{2^2\times 21}{5^2-2^2}=4\,m$

Thus, the height of the embankment is 4 m.

Q18 The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder. ($use\ \pi=\frac{22}{7}$ **)**

Ans:- Let the radius of base and height of the solid right circular cylinder be *r* cm and *h* cm, respectively.

According to the question,

r+*h*=37 .....(1)

Total surface area=1628 sq cm

$2\pi r\left(r+h\right)=1628 \quad .....(2)$

From (1) and (2), we get

$2\pi r(37)=1628$

$\Rightarrow 2\pi r=44$

$\Rightarrow 2\times\frac{22}{7}\times r=44$

$\Rightarrow r=\frac{44\times 7}{2\times 22}$

$\Rightarrow r=7\,cm$

Substituting the value of *r* in (1), we get

$7+h=37$

$\Rightarrow h=30\,cm$

Now,

Volume of the cylinder = πr^2h

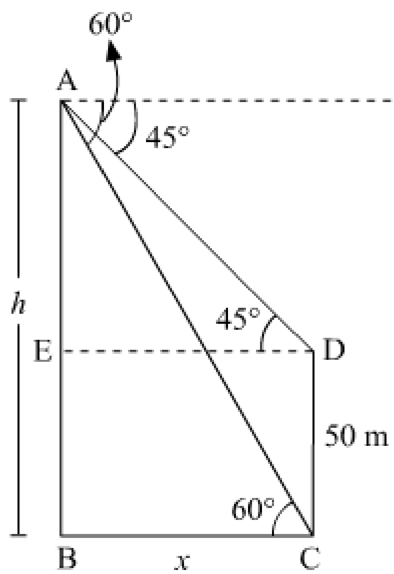
$=\frac{22}{7}\times 7\times 7\times 30$

$=4,620\,cm^3$

Hence, the volume of the cylinder is 4,620 cm^3.

Q19 The angles of depression of the top and bottom of a 50 m high building from the top of a tower are 45° and 60° respectively. Find the height of the tower and the horizontal distance between the tower and the building. $\left(use\ \sqrt{3}=1.73\right)$

Ans:-



Let the height of the tower AB be *h* m and the horizontal distance between the tower and the building BC be *x* m.

So,

AE=(*h*−50) m

$In \Delta AED,$

$$tan 45° = \frac{AE}{ED}$$

$$\Rightarrow 1 = \frac{h-50}{x}$$

$$\Rightarrow x = h-50 \quad .....(1)$$

In

$\Delta ABC,$

$$tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{H}{x}$$

$$\Rightarrow x\sqrt{3} = h \quad .....(2)$$

Using (1) and (2), we get

$x=\sqrt{3}x-50$

$\Rightarrow x\left(\sqrt{3}-1\right)=50$

$\Rightarrow x=\frac{50(\sqrt{3}+1)}{2}=25\times 2.73=68.25m$

Substituting the value of *x* in (1), we get

$68.25=h-50$

$\Rightarrow h=68.25+50$

$\Rightarrow h=118.25m$

Hence, the height of tower is 118.25 m and the horizontal distance between the tower and the building is 68.25 m.

Q20 In a single throw of a pair of different dice, what is the probability of getting (i) a prime number on each dice? (ii) a total of 9 or 11?

Ans:- Total number of outcomes on throwing a pair of dice = 6 × 6 = 36

(*i*) Let E be the event of getting a prime number on each die.

∴ Favourable outcomes = {(2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5)}

Number of favourable outcomes = 9

Now,

$P(E)=\frac{9}{36}=\frac{1}{4}$

Thus, the probability of getting a prime number on each die is $\frac{1}{4}$.

(*ii*) Let F be the event of getting a total of 9 or 11.

∴ Favourable outcomes = {(3, 6), (4, 5), (5, 4), (6, 3), (5, 6), (6, 5)}.

Number of favourable outcomes = 6

Now,

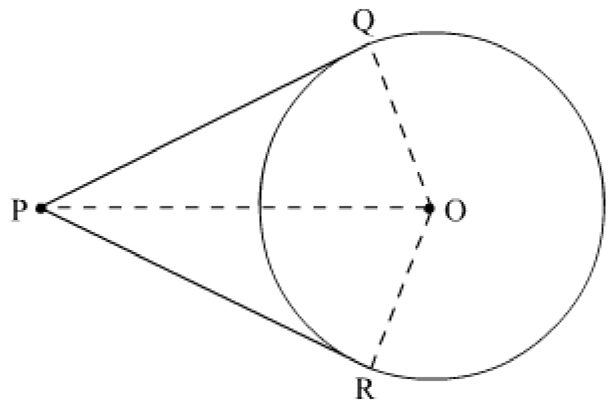
$P(F)=\frac{6}{36}=\frac{1}{6}$ Thus, the probability of getting a total of 9 or 11 is $\frac{1}{6}$.

SECTION D

Q21 A passenger, while boarding the plane, slipped form the stairs and got hurt. The pilot took the passenger in the emergency clinic at the airport for treatment. Due to this, the plane got delayed by half an hour. To reach the destination 1500 km away in time, so that the passengers could catch the

connecting flight, the speed of the plane was increased by 250 km/hour than the usual speed. Find the usual speed of the plane.
What value is depicted in this question?
Ans:- Let the usual speed of the plane be x km/h.
Let the time taken by the plane to reach the destination be t_1.
$$\therefore t_1 = \frac{1500}{x}$$
To reach the destination on time, the speed of the plane was increased to (x + 250)km/h.
$$\therefore t_2 = \frac{1500}{x+250}$$
Given: $t_1 - t_2 = 30\ min$
Now,
$$\frac{1500}{x} - \frac{1500}{x+250} = \frac{30}{60}$$
$$\Rightarrow \frac{1500(x+250-x)}{x(x+250)} = \frac{1}{2}$$
$$\Rightarrow 750000 = x^2 + 250x$$
⇒x2+250x−750000=0
$$\Rightarrow x^2 + 250x - 750000 = 0$$
On solving the equation, we get
x=750
Thus,
Usual speed of the plane = 750 km/h
The value depicted in this question is that of humanity. The pilot has set an example of a good and responsible citizen of the society.

Q22 Prove that the lengths of tangents drawn from an external point to a circle are equal.
Ans:- Given: A circle with centre O, a point P lying outside the circle and PQ and PR as the two tangents
To prove: PQ = PR
Construction: Join OP, OQ and OR.
Proof:



In ΔOQP and ΔORP,

$\angle OQP = \angle ORP = 90^\circ$ (Tangent at any point of a circle is perpendicular to the radius through the point of contact.)

OQ = OR (Radii)

OP = OP (Common)

$\therefore \Delta OQP \cong \Delta ORP$ (By RHS congruency criterion)

⇒ PQ = PR (Corresponding parts of congruent triangles)

Q23 Draw two concentric circles of radii 3 cm and 5 cm. Construct a tangent to smaller circle from a point on the larger circle. Also measure its length.

Ans:- Following are the steps to draw tangents on the given circle:

Step 1

Draw a circle of 3 cm radius with centre O on the given plane.

Step 2

Draw a circle of 5 cm radius, taking O as its centre. Locate a point P on this circle and join OP.

Step 3

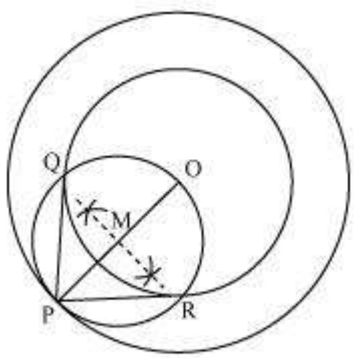
Bisect OP. Let M be the midpoint of PO.

Step 4

Taking M as its centre and MO as its radius, draw a circle. Let it intersect the given circle at points Q and R.

Step 5

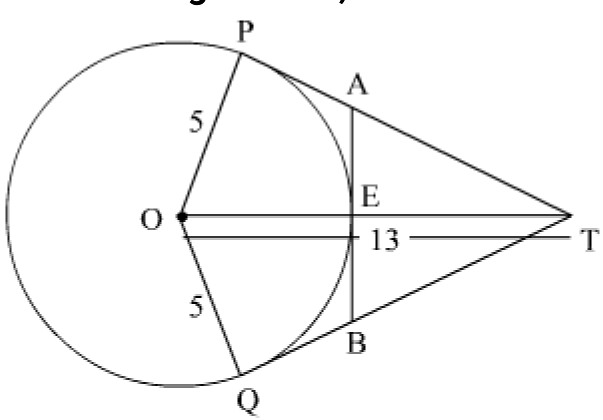
Join PQ and PR. PQ and PR are the required tangents.



It can be observed that PQ and PR are of length 4 cm each.
In $\Delta PQO,$ Since PQ is a tangent,
$\angle PQO = 90°$
PO = 5 cm
QO = 3 cm

Applying Pythagoras theorem in ΔPQO , we obtain
$PQ^2 + QO^2 = PQ^2$
$PQ^2 + (3)^2 = (5)^2$
$PQ^2 + 9 = 25$
$PQ^2 = 25 - 9$
$PQ^2 = 16$
PQ = 4 cm
Hence justified.

Q24 In Fig. 8, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



Ans:- From the given figure, we have

TP = TQ (Two tangents, drawn from an external point to a circle, have equal length.)
and
$\angle TQO = \angle TPO = 90°$ (Tangent to a circle is perpendicular to the radius through the point of contact.)
$In\ \Delta TOQ,$
$QT^2 + OQ^2 = OT^2$
$\Rightarrow QT^2 = 13^2 - 5^2 = 144$
$\Rightarrow QT = 12cm$
Now,
OT − OE = ET = 13 − 5 = 8 cm
Let QB = *x* cm.
∴ QB = EB = *x* (Two tangents, drawn from an external point to a circle, have equal length.)
Also, $\angle OEB = 90°$ (Tangent to a circle is perpendicular to the radius through the point of contact.)
$In\ \Delta TEB,$
$EB^2 + ET^2 = TB^2$
$\Rightarrow x^2 + 8^2 = (12 - x)^2$
$\Rightarrow x^2 + 64 = 144 + x^2 - 24x$
$\Rightarrow 24x = 80$
$\Rightarrow x = \frac{80}{24} = \frac{10}{3}$
$\therefore AB = 2x = \frac{20}{3}\ cm$

Thus, the length of AB is $\frac{20}{3} cm.$

Q25 Find *x* in terms of *a*, *b* and *c*:
$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}, x \neq a, b, c$
Ans:- Consider the given equation:
$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c},$
Taking the LCM and then cross multiplying, we get
$\frac{a(x-b) + b(x-a)}{(x-a)(x-b)} = \frac{2c}{x-c}$

$\Rightarrow (x-c)\ [a(x-b)+b(x-a)]$
$=2c(x-a)(x-b)$
$\Rightarrow (x-c)\ [ax-ab+bx-ab]$
$=2c(x2-bx-ax+ab)$

$$\Rightarrow ax^2-2abx+bx^2-acx+2abc-bcx$$
$$=2cx^2-2bcx-2acx+2abc$$
$$\Rightarrow ax^2+bx^2-2cx^2=2abx-acx-bcx$$

$$\Rightarrow (a+b-2c)x^2=x(2ab-ac-bc)$$
$$\Rightarrow (a+b-2c)x^2-x(2ab-ac-bc)=0$$

$\Rightarrow x[(a+b-2c)x-(2ab-ac-bc)]=0$

$\Rightarrow x=0$ or $(a+b-2c)x-(2ab-ac-bc)=0$

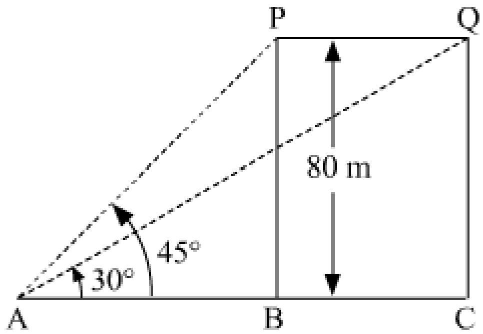
$\Rightarrow x=0$ or $(a+b-2c)x=(2ab-ac-bc)$

$\Rightarrow x=0$ or $x=\frac{(2ab-ac-bc)}{(a+b-2c)}$

Thus, the two roots of the given equation are x = 0 and $\frac{2ab-ac-bc}{a+b-2c}$.

Q26 A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Find the speed of flying of the bird. $\left(Take\ \sqrt{3}=1.732\right)$.

Ans:- Let P and Q be the two positions of the bird, and let A be the point of observation. Let ABC be the horizontal line through A.
Given: The angles of elevations of the bird in two positions P and Q from point A are 45° and 30°, respectively.



$\therefore \angle PAB = 45° \text{ and } \angle QAB = 30°$

Also,

PB = 80 m

In ΔABP, we have

$tan45° = \frac{BP}{AB}$

$\Rightarrow 1 = \frac{80}{AB}$

⇒AB=80 m

In $\Delta ACQ,$ we have

$tan30° = \frac{CQ}{AC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{80}{AC}$

$\Rightarrow AC = 80\sqrt{3}\ m$

∴ PQ = BC = AC − AB

$= 80\sqrt{3} - 80 = 80(\sqrt{3} - 1)m$

So, the bird covers $80(\sqrt{3} - 1)$ m in 2 s.

Thus, speed of the bird is given by

$= \frac{80(\sqrt{3} - 1)}{2} m/s$

$= \frac{40(\sqrt{3} - 1) \times 60 \times 60}{1000} km/h$

=144(1.732−1)km/h

=105.408 km/h.

Q27 `A thief runs with a uniform speed of 100 m/minute. After one minute, a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every

succeeding minute. After how many minutes the policeman will catch the thief.
Ans:- Suppose the policeman catches the thief after *t* minutes.
Uniform speed of the thief = 100 metres/minute
∴ Distance covered by thief in (*t* + 1) minutes = 100 metres/minute × (*t* + 1) minutes = 100 (*t* + 1) metres
Distance covered by policeman in *t* minutes = Sum of *t* terms of an AP with first term 100 and common difference 10

$$=\frac{t}{2}\left[2\times100+(t-1)\times10\right]m$$
$$=t\left[100+5(t-1)\right]$$
$$=t(5t+95)$$
$$=5t^2+95t$$

When the policeman catches the thief,

$$5t^2+95t=100(t+1)$$
$$\Rightarrow 5t^2+95t=100t+100$$
$$\Rightarrow 5t^2-5t-100=0$$
$$\Rightarrow t^2-t-20=0$$

⇒ *t* = − 4 and *t* = 5
∴ *t* = 5 (As *t* cannot be negative)
Thus, the policeman catches the thief after 5 minutes.

Q28 Prove that the area of a triangle with vertices (*t*, *t* −2), (*t* + 2, *t* + 2) and (*t* + 3, *t*) is independent of *t*.
Ans:- Let A(*t*, *t* − 2), B(*t* + 2, *t* + 2) and C(*t* + 2, *t*) be the vertices of the given triangle.
We know that the area of the triangle having vertices
$(x_1,y_1),\ (x_2,y_2)\ and\ (x_3,y_3)$ is

$$\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|.$$

∴ Area of ?ABC $=|\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]|$

$$=|\frac{1}{2}\left[t(t+2-t)+(t+2)(t-t+2)+(t+3)(t-2-t-2)\right]|$$

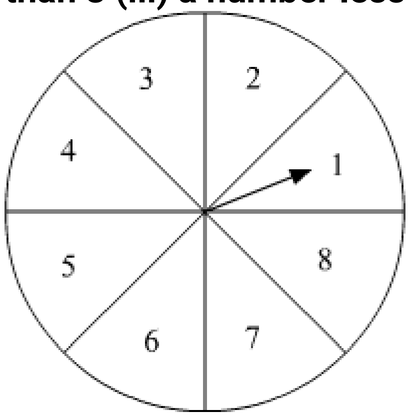
$=|\frac{1}{2}(2t+2t+4-4t-12)|$

$=|-4|$

=4 square units

Hence, the area of the triangle with given vertices is independent of *t*.

Q29 A game of chance consists of spinning an arrow on a circular board, divided into 8 equal parts, which comes to rest pointing at one of the numbers 1, 2, 3, ..., 8 (Fig. 9), which are equally likely outcomes. What is the probability that the arrow will point at (i) an odd number (ii) a number greater than 3 (iii) a number less than 9.



Ans:- Arrow can come to rest at any of the numbers 1, 2, 3, 4, 5, 6, 7 and 8.

Total number of events = 8

(i) There are four odd numbers 1, 3, 5 and 7.

Probability that the arrow will point at an odd number is given by

P (Arrow point at odd number)

$=\frac{4}{8}=\frac{1}{2}$

(ii) There are five numbers greater than 3, that is, 4, 5, 6, 7 and 8.

Probability that the arrow will point at a number greater than 3 is given by

P (Arrow point at a number greater than 3)

$=\frac{5}{8}$

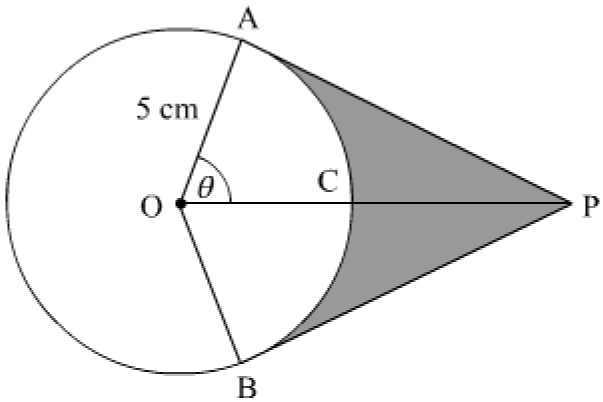
(iii) All the numbers are less than 9.

Probability that the arrow will point at a number less than 9 is given by

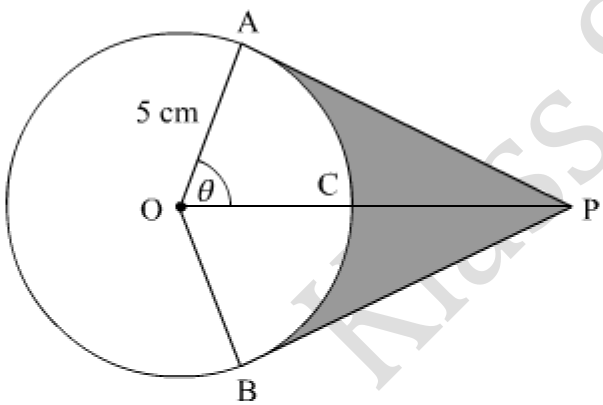
P (Arrow point at a number less than 9)

$=\frac{8}{8}=1$

Q30 An elastic belt is placed around the rim of a pulley of radius 5 cm. (Fig. 10) From one point C on the belt, the elastic belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from the point O. Find the length of the belt that is still in contact with the pulley. Also find the shaded area. $\left(use\ \pi = 3.14\ and\ \sqrt{3} = 1.73\right)$



Ans:-



Given: OA = 5 cm
OP = 10 cm
We know that the tangent at any point of a circle is perpendicular to the radius through the point of contact.
Therefore, ?OAP is a right-angled triangle.
$\Rightarrow \angle OAP = 90°$
Now,
$OP^2 = OA^2 + AP^2$
$\Rightarrow 10^2 = 5^2 + AP^2$
$\Rightarrow AP^2 = 75$
$\Rightarrow AP = 5\sqrt{3}\ cm$
Also,

$$cos\theta = \frac{OA}{OP} = \frac{5}{10}$$
$$\Rightarrow cos\theta = \frac{1}{2}$$
$$\Rightarrow \theta = 60^\circ$$
Now, $\angle AOP = \angle BOP = 60^\circ \quad (\because \Delta OAP \cong \Delta OBP)$
$$\Rightarrow \angle AOB = 120^\circ$$
Length of the belt still in contact with the pulley = Circumference of the circle − Length of the arc ACB
$$= 2\times 3.14\times 5 - \frac{120^\circ}{360^\circ}\times 2\times 3.14\times 5$$
$$= 2\times 3.14\times 5\times\left(1-\frac{1}{3}\right)$$
$$= 2\times 3.14\times 5\times\frac{2}{3}$$
=20.93 cm (Approx.)
Now,

Area of $\Delta OAP \ = \frac{1}{2}\times AP\times OA \ = \frac{1}{2}\times 5\sqrt{3}\times 5 = \frac{25\sqrt{3}}{2} cm^2$

Similarly,

Area of $\Delta OBP \ = \ \frac{25\sqrt{3}}{2} cm^2$

∴ Area of ?OAP + Area of
$\Delta OBP \ = \ 25\sqrt{3}\ cm^2 = 25\times 1.73 = 43.25 cm^2$

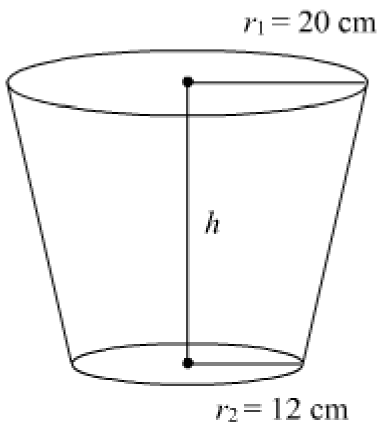
Area of sector OACB $= \frac{120^0}{360^\circ} \times 3.14\times(5)^2$

$$= \frac{1}{3}\times 3.14\times 25 = 26.17\, cm2 (Approx.)$$
∴ Area of the shaded region = (Area of ?OAP + Area of ?OBP) − Area of the sector OACB
$$= \ 43.25\, cm^2 - \ 26.17\, cm^2$$
$$= \ 17.08\, cm^2 \quad (Approx.)$$

Q31 A bucket open at the top is in the form of a frustum of a cone with a capacity of $12308.8\ cm^3$. The radius of the top and bottom circular ends are 20

cm and 12 cm, respectively. Find the height of the bucket and the area of metal sheet used in making the bucket. $(use\ \pi = 3.14)$

Ans:- Consider the following figure:



Given: Volume of the frustum is $12308.8\ cm^3$.
Radius of the top and bottom are $r_1 = 20\ cm\ and\ r_2 = 12\ cm,$ respectively.
Volume of the frustum is given by

$$V = \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$12308.8 \times 3 = \pi h\left(20^2 + 12^2 + 20 \times 12\right)$$

$$12308.8 \times 3 = \pi h\left(400 + 144 + 240\right)\ 12308.8 \times 3 = \pi h\left(784\right)$$

$$\frac{12308.8 \times 3}{3.14 \times 784} = h$$

$$\frac{3920 \times 3}{784} = h$$

15 cm=*h*
Hence, height of the frustum is 15 cm.
Now,
Metal sheet required to make the frustum = Curved surface area + Area of the base of the frustum
Curved surface area of the frustum

$= \pi\left(r_1 + r_2\right)l,$ where $l = \sqrt{h^2 + \left(r_1 - r_2\right)^2}$

$$l = \sqrt{15^2 152 + \left(20 - 12\right)^2}$$

$$= \sqrt{225 + 64} = \sqrt{289} = 17\ cm$$

Curved surface area of the frustum

$= \pi(20+12)17$

$= 544 \times 3.14$

$= 1708.16\, cm^2$

Area of the base $= \pi 12^2 = 144 \times 3.14 = 452.16\, cm^2$

∴ Metal sheet required to make the frustum=1708.16+452.16=2160.32 cm^2